Social Preferences under Uncertainty*

Equality of Opportunity vs. Equality of Outcome

Kota SAITO †

Assistant Professor of Economics

California Institute of Technology

March 29, 2012

Abstract

This paper introduces a model of inequality aversion that captures a preference for equality of ex-ante expected payoff relative to a preference for equality of ex-post payoff by a single parameter. On deterministic allocations, the model reduces to the model of Fehr and Schmidt (1999). The model provides a unified explanation for recent experiments on probabilistic dictator games and dictator games under veil of ignorance. Moreover, the model can describe experiments on a preference for efficiency, which seem inconsistent with inequality aversion. We also apply the model to the optimal tournament. Finally, we provide a behavioral foundation of the model.

*This paper was originally circulated as K. Saito (January 25, 2008) "Social Preference under Uncertainty," the University of Tokyo, COE Discussion Papers, No. F-217 (http://www2.e.u-tokyo.ac.jp/cemano/research/DP/dp.html). The original paper on inequality aversion was first presented on January 16, 2007 at a micro-workshop seminar at the University of Tokyo.

†E-mail: saito@caltech.edu. I am indebted to my adviser, Eddie Dekel, for continuous guidance, support, and encouragement. I am grateful to Colin Camerer, Federico Echenique, Jeff Ely, Ernst Fehr, Faruk Gul, Pietro Ortoleva, Marciano Siniscalchi, and Leeat Yariv for discussions that have led to improvement of the paper. I would like to thank my former adviser, Michihiro Kandori, for his guidance when I was at the University of Tokyo. I would also like to thank Elchanan Ben-Porath, Kenneth Binmore, Soo Hong Chew, David Dillenberger, Thibault Gajdos, Itzhak Gilboa, Edi Karni, David Levine, Shachar Kariv, Fabio Maccheroni, Massimo Marinacci, Philippe Mongin, John E. Roemer, Jacob S. Sagi, Uzi Segal, and seminar participants at the University of Tokyo, Northwestern University, California Institute of Technology, and Risk, Uncertainty, and Decision (RUD, 2010) in Paris. I gratefully acknowledge financial support from the Center for Economic Theory at the Economics Department of Northwestern University.

Keywords: behavioral economics; social preferences; other-regarding preferences; inequality aversion; preference for efficiency; behavioral foundation.

JEL Classification Numbers: C01, D01, D03.

1 Introduction

Consider two drifters X and Y on a solitary island far out at sea. Unexpectedly, X finds a plank with which only one of them can escape from the island. Y proposes "We should use a fifty-fifty lottery to decide which of us gets to use the plank." Only X knows where the plank is, so he gets the plank by rejecting Y's offer. However, rejecting the offer makes X feel guilty. From an ex-ante viewpoint, the lottery gives each drifter an equal probability of getting the plank. However, from an ex-post viewpoint, the lottery may make X feel envy if Y uses the plank. Should X accept Y's offer ?

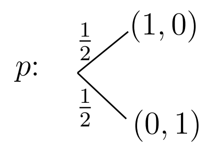


Figure 1: The fifty-fifty lottery (1 denotes getting the plank and 0 denotes not getting it)

This example shows that an individual has a preference for two distinct notions of equality: *equality of opportunity* (i.e., equality of ex-ante expected payoff) and *equality of outcome* (i.e., equality of ex-post payoff). The individual could face a trade-off between them, especially when his ex-post payoff may be inferior to others' ex-post payoff.

In the literature on social preferences, many theories have been proposed on inequality aversion of an individual but most of them study deterministic environments.[1] As Fudenberg and Levine (2011) point out, "leading theories of outcome-based social preferences, . . . , fail to reflect concerns for ex-ante fairness."[2] In social choice literature, several papers have studied the trade-off between the two notions of equality.[3] However, they focus on an impartial social planner; much less attention has been devoted to the study of a partial individual who might feel envy or guilt, which seems more relevant to recent experimental

[1]See Camerer (2003) and Fehr and Schmidt (2005) for surveys on models of inequality aversion in deterministic environments.

[2]Ex-ante fairness corresponds to what we call equality of opportunity

[3]See Diamond (1967), Machina (1989), and Gajdos and Maurin (2004) for examples.

evidence that shows equality of opportunity is often a concern for an individual. For instance, in experiments on probabilistic dictator games, where subjects were asked to allocate probabilities to win an indivisible good with a passive recipient, Karni et al. (2008), Bohnet et al. (2008), Kircher et al. (2009), and Bolton and Ockenfels (forthcoming) found that a considerable number of subjects shared the probability to win equally.

The purpose of this paper is to provide a step toward filling this gap by proposing a model of inequality aversion of an individual under risk that captures both a preference for equality of opportunity and a preference for equality of outcome. We call it the *expected inequality-averse (EIA)* model. We show that the EIA model is consistent with various recent experimental evidence. Then, we apply the model to investigate the optimal tournament, where agents are inequality-averse. Finally, we provide a behavioral foundation for the model.

In the EIA model, the decision maker's preference is represented by the following function: For a lottery p on allocations $(x_1, ..., x_n)$ of material payoffs among n individuals,

$$V(p) = \delta\ U\big(E_p(x_1), ..., E_p(x_n)\big) + (1-\delta)\ E_p\big(U(x_1, ..., x_n)\big), \tag{1}$$

where $\delta \in [0, 1]$ and E_p is the expectation operator with respect to p.[4] In addition, U is the model of Fehr and Schmidt (1999): $U(y_1, ..., y_n) = y_1 - \sum_{i=2}^{n}(\alpha_i \max\{y_i - y_1, 0\} + \beta_i \max\{y_1 - y_i, 0\})$, where y_1 denotes the payoff of the decision maker and y_i denotes the payoff of individual i. U captures inequality aversion as follows: for each $i \neq 1$, the term multiplied α_i captures the disutility of *envy* when the decision maker's payoff is smaller than individual i's payoff; the term multiplied β_i captures the disutility of *guilt* when his payoff is larger than individual i's payoff. All of the parameters α, β, and δ are unique and can be identified experimentally.

In the EIA model, the first term represents a preference for equality of opportunity because the term captures inequality aversion in the ex-ante expected payoffs (i.e., $(E_p(x_1), ..., E_p(x_n))$) and, in contrast, the second term represents a preference for equality of outcome because

[4] Let p be a lottery that yields an allocation $(x_1^s, ..., x_n^s)$ with probability α_s for each $s \in \{1, ..., m\}$. Then, $E_p(x_i) = \alpha_1 x_i^1 + \cdots + \alpha_m x_i^m$ and $E_p\big(U(x_1, ..., x_n)\big) = \alpha_1 U(x_1^1, ..., x_n^1) + \cdots + \alpha_m U(x_1^m, ..., x_n^m)$.

the term captures inequality aversion in the ex-post payoffs (i.e., $(x_1, ..., x_n)$). Therefore, the relative weight δ between the first and the second terms captures a preference for equality of opportunity relative to a preference for equality of outcome.

To demonstrate the tractability of the EIA model, reconsider the example of the two drifters. Assume that X's preference is represented by the EIA model. Then, X's utility of the fifty-fifty lottery (i.e., p in Figure 1) is

$$V(p) = \delta\frac{1}{2} + (1-\delta)\Big[\frac{1}{2}(-\alpha) + \frac{1}{2}(1-\beta)\Big]. \tag{2}$$

The lottery p gives the equal expected payoff $\frac{1}{2}$ to both, which is captured by the first term. However, ex-post allocations are not equal, so X could suffer from either envy when Y gets the plank or guilt when X gets the plank with equal probability. This is captured by the second term. Therefore, X accepts the lottery p (i.e., $V(p) > V(1,0)$) if and only if the weight on equality of opportunity is high enough, or $\delta > \dfrac{1+\alpha-\beta}{\alpha+\beta}$.

Notice that this example of the two drifters is similar in spirit to the experiments on probabilistic dictator games, where subjects are asked to allocate probabilities to win an indivisible good, such as the plank in the example. By finding conditions on the parameters in a similar way, we show that the EIA model is consistent with the experiments. We also show that the EIA model is consistent with an experiment by Kariv and Zame (2009) on dictator games under a veil of ignorance.

Then, we apply the EIA model to describe experimental evidence for a preference for *efficiency* (i.e., maximizing the sum of payoffs across individuals), which seems inconsistent with inequality aversion. For example, Charness and Rabin (2002) found that in a dictator game, almost 50% of 80 subjects preferred an efficient but unequal allocation (where the dictator received 375 points and the receiver received 750 points) to an equal but inefficient allocation (where both subjects received 400 points).[5] However, the EIA model can describe both a preference for efficiency and inequality aversion consistently by taking into account the fact that in the experiments, each subject makes a decision as if he were a dictator, but

[5] See Engelmann and Strobel (2004, 2006) for other examples.

the actual roles (i.e., dictator or recipient) are assigned at random after the decision. Under this *role-assignment risk*, choosing an efficient but unequal allocation can be compatible with equality of opportunity because other subjects also have the same chance to choose the same unequal allocation.

Moreover, we apply the EIA model to the optimal tournament of a firm in which employees are inequality-averse among them. We show that in contrast to the standard model, when $\delta < 1$ the firm cannot make the wage differences between the winner and the loser large enough to implement the employees' efficient effort because of their inequality aversion and participation constraints. However, as the employees' concerns about equality of opportunity become larger (i.e., δ approaches 1), the firm can make the wage differences larger to implement the efficient effort while keeping the participation constraints. This is because in any equilibrium both employees exert the same effort level so that both have the equal opportunity to be the winner. Hence, as δ approaches 1, the employees' disutilities of inequality aversion vanish.

We conclude the paper by providing behavioral conditions that characterize the EIA model.[6] We introduce a novel condition, the *dominance* condition, which states that if p is preferred to q in both equality of opportunity and equality of outcome, then p should be preferred to q. In addition to this key condition, we also assume a *weaker* version of the (von Neumann-Morgenstern risk) independence axiom because the original independence axiom implies no preference for equality of opportunity (i.e., the special case of the EIA model where $\delta = 0$). This implication is consistent with Fudenberg and Levine's (2011) main result that the original independence axiom is incompatible with a preference for equality of opportunity.[7]

[6]There are two related axiomatic papers. Rohde (2010) characterized the model of Fehr and Schmidt (1999). Since Rohde (2010) studied a deterministic environment, she does not consider the trade-off between the two notions of equality, which is the main issue in our paper. Independently of Rohde (2010), we obtained the characterization of the model of Fehr and Schmidt. Subsequently and independently to our paper, Borah (2011) characterized a preference for procedural fairness. In his model, the utility of an ex-post allocation realized through a lottery is a weighted sum of the utility of the ex-post allocation and the utility of the marginal distribution of the lottery. (Our paper has circulated since January 2007. A later version of the paper has been available as COE Discussion Paper No. F-217 at the University of Tokyo (http://www2.e.u-tokyo.ac.jp/cemano/ research/ DP/dp.html) since January 2008.)

[7]Based on this result, Fudenberg and Levine (2011) claim that "leading theories of outcome-based social preferences for fairness, those of Fehr and Schmidt (1999), Bolton and Ockenfels (2000), Charness and

The rest of the paper is organized as follows: In Section 2 we introduce the EIA model formally and show its uniqueness. In Section 3 we show the consistency of the EIA model with the the aforementioned experiments. In Section 4 we apply the EIA model to the optimal tournament. In Section 5 we provide a behavioral foundation: a characterization of the EIA model is in Section 5.1 and a characterization of the parameters is in Section 5.2. Finally in Section 6, we provide a conclusion. We provide proofs in the text in Sections 3, 4, and 5.2. The other proofs are in the appendix.

2 The EIA Model

We assume that the set of (material) payoffs is $\mathbb{R}$. Let $I = \{1, ..., n\}$ be the set of individuals and let 1 denote the decision maker. A vector $\boldsymbol{x} = (x_1, ..., x_n)$ in $\mathbb{R}^n$ is called an *allocation* of payoffs among the individuals. The set of probability distributions over the allocations with finite supports is denoted by $\Delta(\mathbb{R}^n)$. An element in $\Delta(\mathbb{R}^n)$ is called a *lottery* on allocations. For any $p \in \Delta(\mathbb{R}^n)$, E_p is the expectation operator with respect to p.[8]

Definition: An *expected inequality-averse (EIA)* model with a triple (α, β, δ) of vectors $\alpha, \beta \in \mathbb{R}_+^{n-1}$ and $\delta \in [0, 1]$ is a function V on $\Delta(\mathbb{R}^n)$ such that

$$V(p) = \delta\ U\big(E_p(\boldsymbol{x})\big) + (1-\delta)\ E_p\big(U(\boldsymbol{x})\big),$$

where $U(\boldsymbol{y}) = y_1 - \sum_{i=2}^{n}(\alpha_i \max\{y_i - y_1, 0\} + \beta_i \max\{y_1 - y_i, 0\})$. If a preference is represented by V, the preference is called an *EIA preference*.[9]

Proposition 1 *The following two statements are equivalent:*
(i) Two EIA models with (α, β, δ) *and* $(\alpha', \beta', \delta')$ *represent the same preference on* $\Delta(\mathbb{R}^n)$.
(ii) $(\alpha, \beta) = (\alpha', \beta')$ *and if* $(\alpha, \beta) \neq \mathbf{0}$ *then* $\delta = \delta'$.

In the case of interest, where the decision maker cares about inequality (i.e., where

Rabin (2002), Cox and Sadiraj (2004), and Andreoni and Miller (2002) all fail to reflect concerns for ex-ante fairness because they are expected utility theories."

[8]If p is a lottery that yields allocation $\boldsymbol{x}^s = (x_1^s, , ..., x_n^s)$ with probability α_s for each $s \in \{1, ..., m\}$, then $E_p(\boldsymbol{x}) = \alpha_1 \boldsymbol{x}^1 + \cdots + \alpha_m \boldsymbol{x}^m = (\alpha_1 x_1^1 + \cdots + \alpha_m x_1^m, ..., \alpha_1 x_n^1 + \cdots + \alpha_m x_n^m)$. In addition, for any function $f : \mathbb{R}^n \to \mathbb{R}$, $E_p(f(\boldsymbol{x})) = \alpha_1 f(\boldsymbol{x}^1) + \cdots + \alpha_m f(\boldsymbol{x}^m)$.

[9]The representation can be extended to allow for risk aversion.

$(\alpha, \beta) \neq \mathbf{0}$), the triple of parameters (α, β, δ) is unique.

3 Consistency with Experiments

We focus on experiments that involve risks because the EIA model reduces to the model of Fehr and Schmidt (1999) on the deterministic allocations. It is well known that the model of Fehr and Schmidt (1999) can describe various experiments that do not involve risks. Moreover, we focus on stochastic versions of dictator games because these games provide the cleanest test for the model.[10]

To describe the risks involved in the experiments, we define a *probability mixture* to be a lottery on allocations and denote the mixture by $\oplus$ as follows:

Definition: For all $\alpha \in [0,1]$ and $p, q \in \Delta(\mathbb{R}^n)$, $\alpha p \oplus (1-\alpha) q \in \Delta(\mathbb{R}^n)$ is a lottery on allocations such that $(\alpha p \oplus (1-\alpha)q)(\boldsymbol{x}) = \alpha p(\boldsymbol{x}) + (1-\alpha) q(\boldsymbol{x}) \in [0,1]$ for each $\boldsymbol{x} \in \mathbb{R}^n$.[11]

For example, p in Figure 1 corresponds to probability mixture $\frac{1}{2}(1,0) \oplus \frac{1}{2}(0,1)$.

3.1 Probabilistic Dictator Games

First, we discuss the experimental results in *probabilistic dictator games*, where dictators allocate probabilities to win an indivisible good. In such experiments, it is observed that a substantial fraction of dictators shared the probability of winning. For example, Kircher et al. (2009) asked 60 subjects to choose among receiving 1 euro, donating 1 euro to Red Cross, or randomizing between the two with equal probability. Kircher et al. (2009) found that 25% of the subjects chose the equal randomization.[12] See Karni et al. (2008), Bohnet et al. (2008), and Bolton and Ockenfels (forthcoming) for similar experimental evidence.

To see that this experimental finding is consistent with the EIA model, consider a dictator whose preference is represented by the EIA model. The dictator chooses a probability $r \in [0,1]$ that he receives the indivisible good ex post. Since the recipient receives the good with the rest of the probability $1-r$, the dictator obtains probability mixture $p(r) =$

[10]The dictator games perfectly fit the framework of a single-person decision-making problem and do not involve non-distributional concerns such as reciprocity.

[11]For degenerate lotteries on allocations, we write $\alpha \boldsymbol{x} \oplus (1-\alpha)\boldsymbol{y} \in \Delta(\mathbb{R}^n)$, instead of $\alpha \delta_{\boldsymbol{x}} \oplus (1-\alpha)\delta_{\boldsymbol{y}}$.

[12]In a similar experiment with 92 subjects, Kircher et al. (2009) also found that almost 30% of the subjects randomized with equal probability.

$r(1,0) \oplus (1-r)(0,1)$.[13] Thus, the utility by choosing r is [14]

$$V\big(p(r)\big) = \begin{cases} \delta\big(r - \alpha(1-2r)\big) + (1-\delta)\big(r(1-\beta) + (1-r)(-\alpha)\big) & \text{if } r \le \frac{1}{2}, \\ \delta\big(r - \beta(2r-1)\big) + (1-\delta)\big(r(1-\beta) + (1-r)(-\alpha)\big) & \text{if } r \ge \frac{1}{2}. \end{cases} \tag{3}$$

Under the commonly used assumption that disutility from envy is larger than that from guilt (i.e., $\alpha > \beta$), the optimal probability r^* (i.e., the maximizer of (3)) is characterized as follows.[15]

Proposition 2 *Suppose that $\alpha > \beta$. The optimal probability is*

$$r^* = \begin{cases} \dfrac{1}{2} & \textit{if } \delta \ge \dfrac{1+\alpha-\beta}{\alpha+\beta}, \\ 1 & \textit{if } \delta \le \dfrac{1+\alpha-\beta}{\alpha+\beta}. \end{cases}$$

Proof: Since $\alpha > \beta$, it is easy to see that $r \le \frac{1}{2}$ is not optimal. For any $r \in [\frac{1}{2}, 1]$, define $f(r) \equiv V(p(r)) = (\alpha+\beta)\delta - \alpha + (1+\alpha-\beta-\delta(\alpha+\beta))r$. By the linearity of f, the candidates for the maximizers are $r = \frac{1}{2}$ or $r = 1$. In addition, $f(\frac{1}{2}) > f(1) \Leftrightarrow \frac{1}{2}[1-(1-\delta)(\alpha+\beta)] > 1-\beta \Leftrightarrow \delta > \frac{1+\alpha-\beta}{\alpha+\beta}$. ■

Therefore, the dictator shares the probability of winning if and only if $\delta \ge \dfrac{1+\alpha-\beta}{\alpha+\beta}$. Given the values of α and β that are often assumed in the literature, the condition is compatible with $\delta \in [0,1]$.[16]

3.2 Dictator Games under a Veil of Ignorance

Next, we investigate Kariv and Zame's (2009) experiments. In their experiments, subjects are asked to divide payoff 1 into x and y such that $x + qy \le 1$, where $q \ge 1$. After the decision, the material payoffs of the decision maker and the recipient are determined

[13] This specification of the allocation is without loss of generality. We can obtain the same result with x and y such that $x > y$, instead of 1 and 0.

[14] By definition, $V(p(r)) = \delta\big(r - \alpha\max\{1-2r, 0\} - \beta\max\{2r-1, 0\}\big) + (1-\delta)\big(r(1-\beta) + (1-r)(-\alpha)\big)$. A direct calculation shows the equation for each case.

[15] The assumption $\alpha > \beta$ is common in the literature. See, for example, Fehr and Schmidt (1999, 2006).

[16] For $\alpha = 1$ and $\beta = 0.6$, which are assumed for 30 percent of the population by Fehr and Schmidt (1999), the condition is equivalent to $\delta \ge 0.875$. For $\alpha = 2$ and $\beta = 0.6$, which are assumed for 10 percent of the population by Fehr and Schmidt (1999), the condition is equivalent to $\delta \ge 0.93$.

to be x or y with probability $\frac{1}{2}$. Thus, the decision maker obtains probability mixture $\frac{1}{2}(x,y) \oplus \frac{1}{2}(y,x)$. In other words, the subjects are required to decide an allocation under a *veil of ignorance.*

Kariv and Zame (2009) found that the mode of the distribution of x is around $x = \frac{1}{1+q}$, which corresponds to dividing the payoff equally. The distribution falls off sharply from the mode but has a mass (about 7%) at $x = 1$. To see that the EIA model can describe these two major choices $x = \frac{1}{1+q}$ and $x = 1$, note that the utility of the fifty-fifty probability mixture is[17]

$$V\Big(\frac{1}{2}(x,y) \oplus \frac{1}{2}(y,x)\Big) = \frac{1}{2}\Big[(x+y) - (1-\delta)(\alpha+\beta)|x-y|\Big]. \tag{4}$$

Hence, the optimal allocation (x^*, y^*) (i.e., the maximizer of (4)) is characterized as follows:

Proposition 3 *The optimal allocation is*

$$(x^*, y^*) = \begin{cases} (1,0) & \textit{if } \delta \geq 1 - \dfrac{q-1}{(\alpha+\beta)(q+1)}, \\ \Big(\dfrac{1}{1+q}, \dfrac{1}{1+q}\Big) & \textit{if } \delta \leq 1 - \dfrac{q-1}{(\alpha+\beta)(q+1)}. \end{cases}$$

Proof: Since V is monotonic on equal allocations, the budget constraint is binding in the optimal allocation.[18] By the substitution of $x = 1 - qy$, the utility by choosing y is $V(\frac{1}{2}(1-qy, y) \oplus \frac{1}{2}(y, 1-qy)) = \frac{1}{2}[1 + (1-q)y - (1-\delta)(\alpha+\beta)|1-(1+q)y|]$. Define $f(y) \equiv V(\frac{1}{2}(1-qy,y) \oplus \frac{1}{2}(y,1-qy))$. Since $q \geq 1$, $y > \frac{1}{1+q}$ is not optimal. By the linearity of f, the candidates for the maximizers are $y = 0$ or $y = \frac{1}{1+q}$. In addition, $f(0) > f(\frac{1}{1+q}) \Leftrightarrow \frac{1}{2}[1-(1-\delta)(\alpha+\beta)] > \frac{1}{1+q} \Leftrightarrow \delta > 1 - \frac{q-1}{(\alpha+\beta)(q+1)}$. ■

3.3 Preference for Efficiency

Third, we investigate a preference for efficiency under role-assignment risk. Several experimental research has found an evidence for a preference for efficiency, which seems inconsis-

[17]The derivation is as follows: $V(\frac{1}{2}(x,y) \oplus \frac{1}{2}(y,x)) = \delta\frac{1}{2}(x+y) + (1-\delta)[\frac{1}{2}(x - \alpha\max\{y-x,0\} - \beta\max\{x-y,0\}) + \frac{1}{2}(y - \alpha\max\{x-y,0\} - \beta\max\{y-x,0\})] = \frac{1}{2}[(x+y) - (1-\delta)(\alpha+\beta)|x-y|]$.

[18]$V(x,\dots,x) \geq V(y,\dots,y) \Leftrightarrow x \geq y$.

tent with inequality aversion. For example, Charness and Rabin (2002) have found that in a dictator game under role-assignment risk, almost 50% of 80 subjects preferred an efficient but unequal allocation to an equal but inefficient allocation. (See Engelmann and Strobel (2004, 2006) for other examples.)

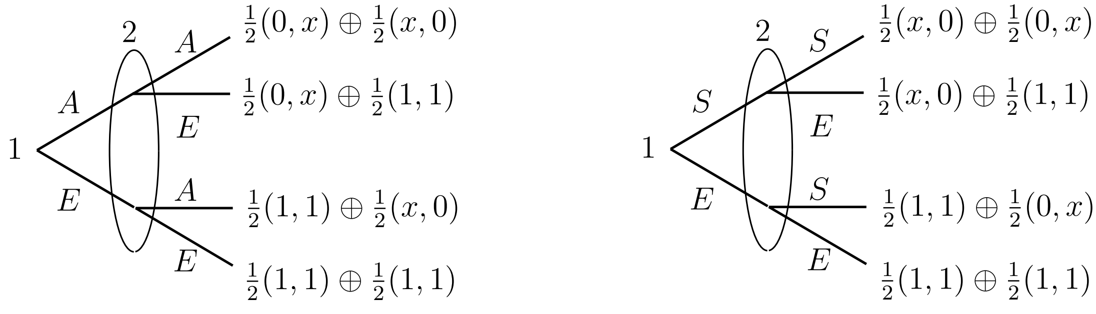
We consider two versions of dictator games where a dictator can choose between an efficient (but unequal) allocation or an equal (but inefficient) allocation: in one version of the dictator game, the efficient allocation is an altruistic allocation and in the other version, the efficient allocation is a selfish allocation. Then, assuming mild assumptions on the parameters, we show that under role-assignment risk, a subject who has an EIA preference can choose the altruistic allocation in one game and the selfish allocation in the other game but without role-assignment risk he always chooses the equal allocation over those efficient allocations.

The games are formally defined as follows. We consider two players 1 and 2 who have EIA preferences. In game I, the players make a choice between an altruistic allocation and an equal allocation under role-assignment risk. If a player chooses the altruistic allocation and becomes a dictator, then his payoff is 0 and and the other player's payoff is $x > 2$. In contrast, if a player chooses an equal allocation and becomes a dictator, then both players' payoffs are 1.[19] Note that since $x > 2$, the altruistic allocation is more efficient than the equal allocation.

Due to role-assignment risk, in fact, each player obtains a fifty-fifty lottery over allocations. For example, if player 1 chooses the altruistic allocation (i.e., $(0, x)$) and player 2 chooses the equal allocation (i.e., $(1, 1)$), then both players obtain probability mixture $\frac{1}{2}(0, x) \oplus \frac{1}{2}(1, 1)$, where the first and the second coordinates of the profiles show the material payoffs of players 1 and 2, respectively. Hence, game I is illustrated in Figure 2, where A denotes choosing the altruistic allocation and E denotes choosing the equal allocation.

Game II is the same as game I except for the point that each player can choose a selfish allocation, instead of the altruistic allocation. If a player chooses the selfish allocation and becomes a dictator, then his payoff is $x > 2$ and the other player's payoff is 0. Since $x > 2$,

[19]This specification of allocation is without loss of generality. We can obtain the same result with y instead of 1 and z instead of 0 such that $x > y > z$.



Game I Game II

Figure 2: Dictator games with role-assignment risk

the selfish allocation is more efficient than the equal allocation. Game II is also illustrated in Figure 2, where S denotes choosing the selfish allocation.

Games I' and II' are the same as games I and II, respectively, except for the point that there is no role-assignment risk. Thus, in games I' and II' only one player's choice determines the allocation as illustrated in Figure 3, where player 1 is a dictator for sure.

Game I' Game II'

Figure 3: Dictator games without role-assignment risk

Proposition 4 *Suppose that $\alpha > \beta$ and $1 > \beta$.*[20] *Then, for any payoff x such that*

$$\delta(\alpha+\beta) - \alpha \geq \frac{1}{x} \geq 1 - \beta, \tag{5}$$

the following results hold:[21]

(i) In game I, both subjects choose the altruistic allocation in an equilibrium.

(ii) In game II, both subjects choose the selfish allocation in an equilibrium.

(iii) In games I' and II', choosing the equal allocation is dominant.[22]

[20]The assumptions $\alpha > \beta$ and $1 > \beta$ are common in the literature. See, for example, Fehr and Schmidt (1999, 2006).

[21]Under the assumption that $\beta > \frac{1}{2}$, x that satisfies (5) and $x > 2$ exists if and only if $\delta(\alpha+\beta)-\alpha \geq 1-\beta$.

[22]In (i) and (ii), there is the other symmetric pure equilibrium where both choose the equal allocation.

Proof: By a direct calculation, (iii) holds if $1 > x(1-\beta)$. To show (i) and (ii), we denote by $\Pi_i(s_i|s_j)$ the utility of player i when his strategy is s_i and the strategy of player j is s_j. Then, $\Pi_1(A|A) = V\big(\frac{1}{2}(0,x) \oplus \frac{1}{2}(x,0)\big) = \frac{1}{2}x[1-(1-\delta)(\alpha+\beta)]$, $\Pi_1(E|A) = V\big(\frac{1}{2}(1,1) \oplus \frac{1}{2}(x,0)\big) = \frac{1}{2}[1+x(1-\beta)]$, $\Pi_1(S|S) = V\big(\frac{1}{2}(x,0) \oplus \frac{1}{2}(0,x)\big) = \frac{1}{2}x[1-(1-\delta)(\alpha+\beta)]$, and $\Pi_1(E|S) = V\big(\frac{1}{2}(1,1) \oplus \frac{1}{2}(0,x)\big) = \frac{1}{2}(1-\alpha x)$. Since the game is symmetric across the players, $\Pi_i(A|A) > \Pi_i(E|A)$ if and only if $(\delta(\alpha+\beta)-\alpha)x > 1$. Similarly, $\Pi_i(S|S) > \Pi_i(E|S)$ if and only if $(\delta(\alpha+\beta)-\beta+1)x > 1$. Since $\alpha > \beta$, $(\delta(\alpha+\beta)-\beta+1)x > (\delta(\alpha+\beta)-\alpha)x$. Therefore, (i), (ii), and (iii) hold if $\delta(\alpha+\beta)-\alpha \geq \frac{1}{x} \geq 1-\beta$. ■

As (iii) states, in games I' and II' without role-assignment risk, the dictator always chooses the equal allocation over both the altruistic allocation and the selfish allocation. However with role-assignment risk, the same dictator can choose the altruistic allocation in game I and the selfish allocation in game II. Therefore, if an experimenter did not take into consideration role-assignment risk in these games, the observed behavior would look inconsistent with inequality aversion and consistent only with a preference for efficiency.

To understand this result intuitively, note that *role-assignment risk has an effect similar to that of a veil of ignorance*. Suppose that each of the two players decides to allocate x to himself and y to the other. Then, under role-assignment risk, the utility of each subject is determined as (4); that is,

$$V\Big(\frac{1}{2}(x,y) \oplus \frac{1}{2}(y,x)\Big) \;=\; \frac{1}{2}\Big[(\text{“efficiency”}) - (1-\delta)(\alpha+\beta)(\text{“inequality”})\Big].$$

Hence, the subject's utility from choosing the efficient allocation can be larger than his utility from choosing the equal allocation, provided that he weighs equality of opportunity heavily enough (i.e., δ is large enough) and the other player chooses the same efficient allocation.

Finally, we stress that our explanation is only one possible hypothesis and does not deny the importance of a preference for efficiency. However, in three-person voting games, Bolton

This is not inconsistent with our main claim in this section: with role-assignment risk, the choices of dictators are unstable (i.e., choosing the equal allocation is not dominant and there is an equilibrium where both players choose the efficient allocation); in contrast, without role-assignment risk, the choices are stable (i.e., choosing the equal allocation is dominant).

and Ockenfels (2006) found experimentally that a preference for efficiency is significantly mitigated without role-assignment risk.[23] This evidence would imply that, as Proposition 4 suggests, part of the preference for efficiency under role-assignment risk would be driven by a preference for equality of opportunity.

4 Application to Optimal Tournament

In this section, we investigate the optimal tournament of a firm with two employees. We assume that the employees i and j are inequality-averse between them and their preferences are represented by the EIA model. We show that in contrast to the standard model by Lazer and Rosen (1981), when $\delta < 1$ the firm cannot implement the employees' efficient effort.[24] However, as δ approaches 1, the firm can asymptotically implement the efficient effort.

The firm chooses wage w_1 for the winner and wage w_2 for the loser to maximize the sum of the employees' productions. The employees compete in their quantity of production. The production of employee $k \in \{i, j\}$ is $q_k(e_k) = e_k + \varepsilon_k$, where e_k is his effort level and ε_k is an exogenous random shock. Let G and g be the distribution function and the density function of $\varepsilon_i - \varepsilon_j$. We assume that ε_i and ε_j are independently and identically distributed and $g(0) > 0$. Then $g(x) = g(-x)$.

The probability that employee i wins is $p_i(e_i, e_j) \equiv \Pr\{q_i(e_i) > q_j(e_j)\} = G(e_i - e_j)$. Hence, when the pair of effort levels is (e_i, e_j), employee i obtains probability mixture $p_i(e_i, e_j)(w_1, w_2) \oplus (1 - p_i(e_i, e_j))(w_2, w_1)$. Given (w_1, w_2) and e_j, employee i chooses e_i to maximize $V(p_i(e_i, e_j)(w_1, w_2) \oplus (1 - p_i(e_i, e_j))(w_2, w_1)) - e_i^2$, where e_i^2 is the cost of effort.

Remark 1 *Suppose $\alpha \geq \beta$. Given (w_1, w_2), in any pure strategy equilibrium, both employees choose an identical effort level $e_1 = e_2 = e$ such that $\Delta w g(0)(\delta(1-2\alpha)+(1-\delta)(1-\beta+\alpha)) \leq 2e \leq \Delta w g(0)(\delta(1-2\beta)+(1-\delta)(1-\beta+\alpha))$, where $\Delta w = w_1 - w_2$.*

Proof: First, we show $e_i = e_j$ in any pure strategy equilibrium. Without loss of generality,

[23] Charness and Rabin (2002) and Engelmann and Strobel (2006) also found that, in different games, the percentage of choices of efficient allocations decreases without role-assignment risk, although the decreases are not significant.

[24] Grund and Sliwka (2005) studied the case where $\delta = 0$.

assume $e_i \geq e_j$. Then $p_i \geq \frac{1}{2} \geq p_j$ and the utility of employee i is $\delta(w_2 + p_i\Delta w - \beta(2p_i - 1)\Delta w) + (1-\delta)(w_2 - \alpha\Delta w + p_i\Delta w(1+\alpha-\beta)) - e_i^2$ and the utility of employee j is $\delta(w_2 + p_j\Delta w - \alpha(1-2p_j)\Delta w) + (1-\delta)(w_2 - \alpha\Delta w + p_j\Delta w(1+\alpha-\beta)) - e_j^2$. By the first order conditions, $0 = \delta(\frac{dp_i}{de_i}\Delta w - 2\beta\frac{dp_i}{de_i}\Delta w) + (1-\delta)(\frac{dp_i}{de_i}\Delta w(1+\alpha-\beta)) - 2e_i$ and $0 = \delta(\frac{dp_j}{de_j}\Delta w + 2\alpha\frac{dp_j}{de_j}\Delta w) + (1-\delta)(\frac{dp_j}{de_j}\Delta w(1+\alpha-\beta)) - 2e_j$. By the symmetry, $\frac{dp_i}{de_i} = g(e_i - e_j) = g(e_j - e_i) = \frac{dp_j}{de_j}$. We obtain $2(e_i - e_j) = -2\delta(\alpha+\beta)\Delta w g(e_i - e_j) \leq 0$. Hence, $e_j \geq e_i$. Therefore, in any pure strategy equilibrium, $e_i = e_j$.

Next, we check incentives to deviate from (e, e). Individual i has no incentive to increase his effort at (e, e) if $\delta(g(0)\Delta w - 2\beta g(0)\Delta w) + (1-\delta)(g(0)\Delta w(1-\beta+\alpha)) - 2e \leq 0$. Similarly, he has no incentive to decrease his effort at (e, e) if $\delta(-g(0)\Delta w - 2\alpha g(0)\Delta w) + (1-\delta)(-g(0)\Delta w(1-\beta+\alpha)) + 2e \leq 0$. Given (w_1, w_2), therefore, (e_i, e_j) is a pair of best responses if and only if $\Delta w\ g(0)(\delta(1-2\alpha) + (1-\delta)(1-\beta+\alpha)) \leq 2e_i = 2e_j \leq \Delta w g(0)(\delta(1-2\beta) + (1-\delta)(1-\beta+\alpha))$. ■

Given (w_1, w_2), equilibrium effort levels of both employees must be the same but there are multiple best-response effort levels because of the kink of the EIA model. In the following, we select two equilibria where the effort level is highest (i.e., $2e = \Delta w g(0)(\delta(1-2\beta) + (1-\delta)(1-\beta+\alpha))$) or lowest (i.e., $2e = \Delta w g(0)(\delta(1-2\alpha) + (1-\delta)(1-\beta+\alpha))$).[25] Since higher effort implies higher profit of the firm and lower utility of the employees, the high effort equilibrium is the best equilibrium for the firm and the low effort equilibrium is the best equilibrium for the employees.[26] We show that both equilibria have the same properties in comparative statics with respect to δ.

In an equilibrium, (w_1, w_2, e) must maximize the firm's expected profit: $E[q_i(e, e) + q_j(e, e)] - w_1 - w_2 = 2e - 2w_2 - \Delta w + E(\varepsilon_i) + E(\varepsilon_j)$ subject to the employee's participation constraint:

$$0 \leq V\Big(\frac{1}{2}(w_1, w_2) \oplus \frac{1}{2}(w_2, w_1)\Big) - e^2 = w_2 + \frac{1}{2}\Delta w(1 - (1-\delta)(\alpha+\beta)) - e^2$$

[25]We assume the standard assumption that $\alpha \geq \beta$.

[26]The high effort equilibrium would arise as a focal point through the influence of the firm. The low effort equilibrium would arise as a result of coordination among the employees.

and the incentive rationality constraint: $2e = \Delta w g(0)(\delta(1-2\beta)+(1-\delta)(1-\beta+\alpha))$ in the high effort equilibrium and $2e = \Delta w g(0)(\delta(1-2\alpha)+(1-\delta)(1-\beta+\alpha))$ in the low effort equilibrium, respectively.

Proposition 5 *Let $\frac{1}{2} \geq \alpha \geq \beta$ and $\delta > 0$. In both equilibria, the following results hold:*
Suppose $\alpha+\beta = 0$, then the effort level is efficient (i.e., $\frac{1}{2}$).
Suppose $\alpha+\beta > 0$, then (i) the effort level is smaller than the efficient level if $\delta < 1$, (ii) the effort level is increasing in δ and tends to be efficient as δ approaches 1 (i.e., $e \to \frac{1}{2}$ as $\delta \to 1$), and (iii) the wage differences Δw is increasing in δ.

Proof: In any equilibrium, the participation constraint is satisfied with equality: $w_2(e) = -\frac{1}{2}(1-(1-\delta)(\alpha+\beta))\Delta w(e) + e^2$. In addition, $\Delta w(e)$ is determined by the incentive constraints in both equilibria: $\Delta w(e) = \frac{2e}{\delta(1-2\beta)+(1-\delta)(1-\beta+\alpha)}$ in the high effort equilibrium and $\Delta w(e) = \frac{2e}{\delta(1-2\alpha)+(1-\delta)(1-\beta+\alpha)}$ in the low effort equilibrium. Since $\alpha \geq \beta$, $\Delta w(e)$ is increasing in δ in both equilibria, if e is increasing in δ.

The firm's problem is $\max_e 2e - 2w_2(e) - \Delta w(e) + E(\varepsilon_i) + E(\varepsilon_j)$. The first order condition is $0 = 2 - 4e - (1-\delta)(\alpha+\beta)(\Delta w)'(e)$. Therefore, $e = \frac{1}{2} - \frac{1}{4}(1-\delta)(\alpha+\beta)(\Delta w)'(e)$. Since $(\Delta w)''(e) = 0$, the second order condition is satisfied. Therefore, the equilibrium effort level is $e = \frac{1}{2} - \frac{\alpha+\beta}{2g(0)} \frac{1}{\frac{\delta}{1-\delta}(1-2\beta)+(1-\beta+\alpha)}$ in the high effort equilibrium and $e = \frac{1}{2} - \frac{\alpha+\beta}{2g(0)} \frac{1}{\frac{\delta}{1-\delta}(1-2\alpha)+(1-\beta+\alpha)}$ in the low effort equilibrium. Since $\frac{1}{2} \geq \alpha \geq \beta$, $\frac{\delta}{1-\delta}$ is increasing in δ, and $\frac{\delta}{1-\delta} \to \infty$ as $\delta \to 1$, the results in Proposition 5 hold. ■

As Lazer and Rosen (1981) have shown, if employees are not inequality-averse (i.e., $\alpha = 0 = \beta$), the equilibrium effort is efficient (i.e., $e = \frac{1}{2}$). However, when they are inequality-averse and $\delta < 1$, as (i) states, the equilibrium effort level is smaller than the efficient level. To understand this result intuitively, note that as the incentive constraints in both equilibria show, implementing higher effort requires larger wage differences Δw. The larger wage differences in turn implies a higher disutility caused by the inequality. Since the firm must compensate the disutility by making w_2 higher, it can be too costly to implement the efficient effort level in the equilibria.

However, as Remark 1 shows, in any equilibrium, the employees' effort levels are the same, so that the payoff distribution $\frac{1}{2}(w_1, w_2) \oplus \frac{1}{2}(w_2, w_1)$ is equal in opportunity. Therefore the disutility caused by inequality aversion vanishes as δ increases. Hence, as (iii) states, the firm can make the wage differences larger as δ increases to implement higher effort while keeping the participation constraints. As a result, as (ii) states, the equilibrium effort level tends to be efficient, as δ approaches 1.

The implication of Proposition 5 might suggest an explanation of the divergence in the wage systems between the United States and Western Europe: the wage differences in the United States are larger than those in Western Europe.[27] A recent meta-analysis by Oosterbeek, Sloof, and van de Kuilen (2004) have suggested no significant difference in deterministic inequality aversion (i.e., α and β) between the two regions.[28]

The EIA model could still explain the divergence in the wage systems by taking into account the difference in a preference for equality of opportunity (i.e., δ). One explanation that is consistent with Proposition 5 (iii) is that Americans are more concerned about equality of opportunity than Western Europeans (i.e., δ is higher in the United States than in Western Europe). In other words, inequality of outcome (i.e., larger wage differences) as a result of fair competition is more acceptable in the United States than in Western Europe.[29]

5 Behavioral Foundation

The primitive preference $\succsim$ is on $\Delta(\mathbb{R}^n)$. As usual, $\succ$ and $\sim$ denote the asymmetric and symmetric parts of $\succsim$, respectively.[30]

5.1 Characterization of the EIA Model

The following five conditions characterize the EIA model. The first condition is standard.

Condition (Rationality): $\succsim$ is complete, transitive, continuous, and monotonic in equal allocations (i.e., $(x, ..., x) \succsim (y, ..., y)$ if and only if $x \geq y$).[31]

[27]See Bertola and Ichino (1995) for the evidence.

[28]Oosterbeek, Sloof, and van de Kuilen (2004) found no significant difference in both offered shares and rejection rates between Western Europe and the Eastern United States by conducting a meta-analysis of 37 papers on ultimatum game experiments.

[29]This explanation is also consistent with the empirical evidence that Americans work more than Western Europeans. See Prescott (2004) for the evidence.

[30]For all $x, x' \in \mathbb{R}$, we write $x \succsim x'$ instead of $(x, ..., x) \succsim (x', ..., x')$, when there is no danger of confusion.

[31]The continuity assumed here is von Neumann-Morgenstern continuity with respect to outcome mixtures

The next condition captures envy and guilt respectively. In the condition, we denote by $(x,(y)_{-i})$ the allocation that gives payoff x only to individual i and gives payoff y to the other individuals.

Condition (Inequality Aversion): For all $i \neq 1$, $(0,(0)_{-i}) \succsim (1,(0)_{-i})$ and $(0,(0)_{-i}) \succsim (-1,(0)_{-i})$.

To provide a weaker version of the independence axiom, we define an *outcome mixture* of allocations as follows. Note that we denote outcome mixtures using $+$, not $\oplus$.

Definition: For all $\alpha \in [0,1]$ and $\boldsymbol{x},\boldsymbol{y} \in \mathbb{R}^n$, $\alpha\boldsymbol{x} + (1-\alpha)\boldsymbol{y} \in \mathbb{R}^n$ is an allocation such that $\alpha\boldsymbol{x} + (1-\alpha)\boldsymbol{y} = (\alpha x_1 + (1-\alpha)y_1, ..., \alpha x_n + (1-\alpha)y_n)$.

Inequality-averse preferences may violate the (von Neumann-Morgenstern) independence axiom. For example, a violation often observed in (two-person) dictator games is as follows: $\frac{1}{2}(1,0) + \frac{1}{2}(0,1) \equiv (\frac{1}{2},\frac{1}{2}) \succ (1,0) \succsim (0,1)$.[32] The source of the violation here is the fact that in the mixed allocations, the rankings of the payoffs are opposite (i.e., the other individual is better off in $(0,1)$ and worse off in $(1,0)$ than the decision maker).

We want to assume the independence axiom when such reversals do not occur.

Definition: Two allocations $\boldsymbol{x},\boldsymbol{y} \in \mathbb{R}^n$ are said to be *quasi-comonotonic* if there is no $i \neq 1$ such that $x_i > x_1$ and $y_i < y_1$. (Remember that $1 \in I$ denotes the decision maker.)

If $\boldsymbol{x}$ and $\boldsymbol{y}$ are quasi-comonotonic, then the rankings of payoffs of any individual with respect to the decision maker are not reversed between $\boldsymbol{x}$ and $\boldsymbol{y}$.[33]

Condition (Outcome Quasi-comonotonic Independence): For all $\alpha \in (0,1]$ and $\boldsymbol{x},\boldsymbol{y},\boldsymbol{z} \in \mathbb{R}^n$ such that the pairs $\boldsymbol{x},\boldsymbol{z}$, and $\boldsymbol{y},\boldsymbol{z}$ are each quasi-comonotonic,

---

and probability mixtures. Formally, (i) for all $\boldsymbol{x},\boldsymbol{y},\boldsymbol{z} \in \mathbb{R}^n$, if $\boldsymbol{x} \succ \boldsymbol{y}$ and $\boldsymbol{y} \succ \boldsymbol{z}$, then there exist α and β in $(0,1)$ such that $\alpha\boldsymbol{x} + (1-\alpha)z \succ \boldsymbol{y}$ and $\boldsymbol{y} \succ \beta\boldsymbol{x} + (1-\beta)\boldsymbol{z}$; and (ii) for all $p,q,r \in \Delta(\mathbb{R}^n)$, if $p \succ q$ and $q \succ r$, then there exist α and β in $(0,1)$ such that $\alpha p \oplus (1-\alpha)r \succ q$ and $q \succ \beta p \oplus (1-\beta)r$.

[32] Suppose that $\succsim$ satisfies the independence axiom. Then $(1,0)\succsim(0,1)$ implies $(1,0) \equiv \frac{1}{2}(1,0) + \frac{1}{2}(1,0) \succsim \frac{1}{2}(1,0) + \frac{1}{2}(0,1) \equiv (\frac{1}{2},\frac{1}{2})$. This contradicts with $(\frac{1}{2},\frac{1}{2}) \succ (1,0)$.

[33] The definition is a weaker version of *comonotonicity* provided by Schmeidler (1989). Schmeidler (1989, p. 586) has presented an interpretation of comonotonicity from the viewpoint of a *social planner's inequality aversion* as follows: two allocations $\boldsymbol{x}$ and $\boldsymbol{y}$ are comonotonic if the social rank of *any two individuals* is not reversed between $\boldsymbol{x}$ and $\boldsymbol{y}$. When we focus on an *individual's inequality-averse preferences*, what is relevant to the individual is social rank *with respect to the individual himself*, not the social rank of *any* two individuals.

$$\boldsymbol{x} \succsim \boldsymbol{y} \Leftrightarrow \alpha\ \boldsymbol{x} + (1-\alpha)\boldsymbol{z} \succsim \alpha\boldsymbol{y} + (1-\alpha)\boldsymbol{z}.$$

These three conditions are enough to obtain the model of Fehr and Schmidt (1999) on the restricted domain $\mathbb{R}^n$ (i.e., the set of degenerate lotteries on allocations).[34]

Just as outcome mixtures may change the rankings of ex-post payoffs, probability mixtures may change the rankings of ex-ante *expected* payoffs. However, probability mixtures with an equal allocation $(x, ..., x)$ should not change any rankings. This suggests the following condition:

Condition (Probability Certainty Independence): For all $\alpha \in (0, 1]$, $p, q \in \Delta(\mathbb{R}^n)$, and $x \in \mathbb{R}$,

$$p \succsim q \Leftrightarrow \alpha p \oplus (1-\alpha)(x, ..., x) \succsim \alpha q \oplus (1-\alpha)(x, ..., x).$$

Under the above conditions, for any allocation $\boldsymbol{x}$, it will be shown that there uniquely exists $z \in \mathbb{R}$ such that $(z, ..., z) \sim \boldsymbol{x}$. Such payoff z is denoted by $e(\boldsymbol{x})$ and is called an *equality equivalent*. The value $e(\boldsymbol{x})$ reflects the equality of allocation $\boldsymbol{x}$.

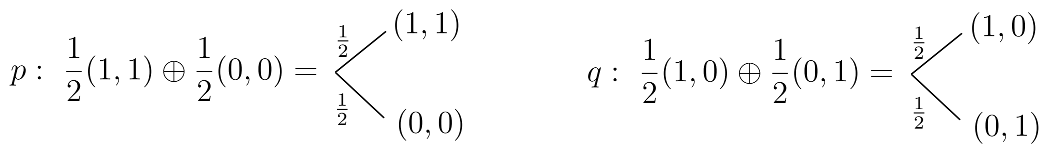
The final key condition, dominance, states that if p is preferred to q in both equality of opportunity and equality of outcome, then p should be preferred to q. To show the condition, we formalize the two notions of equality by using equality equivalents. Let $p = \alpha_1\boldsymbol{x}^1 \oplus \cdots \oplus \alpha_m\boldsymbol{x}^m$. Then, $E_p(\boldsymbol{x}) = (\sum_{i=1}^m \alpha_i x_1^i, ..., \sum_{i=1}^m \alpha_i x_n^i)$ is the expected allocation in p. Therefore $e(E_p(\boldsymbol{x}))$ reflects the *evaluation of* p *in terms of equality of opportunity.* In contrast, since each $e(\boldsymbol{x}^i)$ reflects the equality of outcome $\boldsymbol{x}^i$, $E_p(e(\boldsymbol{x})) = \alpha_1 e(\boldsymbol{x}^1) + \cdots + \alpha_m e(\boldsymbol{x}^m)$ is the expected *evaluation of* p *in terms of equality of outcome.*

Condition (Dominance): For all $p, q \in \Delta(\mathbb{R}^n)$,

$$\left\{ \begin{array}{lrcl} \text{(i)} & e(E_p(\boldsymbol{x})) & \geq & e(E_q(\boldsymbol{x})) \\ \text{(ii)} & E_p(e(\boldsymbol{x})) & \geq & E_q(e(\boldsymbol{x})) \end{array} \right) \Rightarrow p \succsim q.$$

To demonstrate how to use the dominance condition, let $p = \frac{1}{2}(1, 1) \oplus \frac{1}{2}(0, 0)$ and $q = \frac{1}{2}(1, 0) \oplus \frac{1}{2}(0, 1)$. On one hand, $E_p(\boldsymbol{x}) = (\frac{1}{2}, \frac{1}{2}) = E_q(\boldsymbol{x})$, so that $e(E_p(\boldsymbol{x})) = \frac{1}{2} = e(E_q(\boldsymbol{x}))$. Hence, p and q are indifferent in equality of opportunity. On the other hand, to evaluate the

[34] See Lemma 1 in the appendix for the statement.



(i) $e(E_p(\boldsymbol{x})) = e(\frac{1}{2}, \frac{1}{2})$ $\qquad$ $e(E_q(\boldsymbol{x})) = e(\frac{1}{2}, \frac{1}{2})$

(ii) $E_p(e(\boldsymbol{x})) = \frac{1}{2}e(1,1) + \frac{1}{2}e(0,0)$ $\qquad$ $E_q(e(\boldsymbol{x})) = \frac{1}{2}e(1,0) + \frac{1}{2}e(0,1)$

Figure 4: Dominance

lotteries in equality of outcome, suppose that the decision maker is strictly inequality-averse (i.e., $(1,1) \succ (1,0)$ and $(0,0) \succ (0,1)$), so that $1 > e(1,0)$ and $0 > e(0,1)$. Since $e(1,1) = 1$ and $e(0,0) = 0$, it follows that $E_p(e(\boldsymbol{x})) = \frac{1}{2}e(1,1) + \frac{1}{2}e(0,0) = \frac{1}{2} > \frac{1}{2}e(1,0) + \frac{1}{2}e(0,1) = E_q(e(\boldsymbol{x}))$. Hence, p is preferred to q in equality of outcome. Therefore, the dominance condition implies that the decision maker should prefer p to q.[35]

The above five conditions are equivalent to an EIA model. Formally,

Theorem: $\succsim$ *satisfies Rationality, Inequality Aversion, Outcome Quasi-comonotonic Independence, Probability Certainty Independence, and Dominance if and only if there exists a triple* (α, β, δ) *of vectors* $\alpha, \beta \in \mathbb{R}_+^{n-1}$ *and* $\delta \in [0,1]$ *such that* $\succsim$ *is represented by*

$$V(p) = \delta \; U\big(E_p(\boldsymbol{x})\big) + (1-\delta) \; E_p\big(U(\boldsymbol{x})\big),$$

where $U(\boldsymbol{y}) = y_1 - \sum_{i=2}^{n}(\alpha_i \max\{y_i - y_1, 0\} + \beta_i \max\{y_1 - y_i, 0\})$.

5.2 Characterizations of Parameters

We can identify the parameters easily by experiments: α and β are respectively identified by finding α_i and β_i such that

$$(-\alpha_i, ..., -\alpha_i) \sim (1, (0)_{-i}) \text{ and } (-\beta_i, ..., -\beta_i) \sim (-1, (0)_{-i}), \tag{6}$$

[35] In contrast to the dominance condition, the reversal of order condition proposed by Anscombe and Aumann (1963) does not take equality of outcome into consideration: the condition implies that if $e(E_p(\boldsymbol{x})) = e(E_q(\boldsymbol{x}))$ then $p \sim q$.

for each $i \in I \setminus \{1\}$.[36] After identifying α and β, we can identify δ by finding c such that

$$(c, ..., c) \sim \frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i}), \tag{7}$$

for some $i \in I \setminus \{1\}$.[37] Then, by the representation, $\delta = 1 + \dfrac{2c}{\alpha_i + \beta_i}$. [38]

In the following, we characterize the special cases of the EIA model with $\delta = 1$ and $\delta = 0$.[39] The next condition characterizes the special with $\delta = 1$. The condition states that allocation $(0, ..., 0)$ is indifferent to a probability mixture whose ex-ante expected allocation is $(0, ..., 0)$.

Condition (Indifference to Timing of Mixture): For all $i \neq 1$,

$$(0, ..., 0) \sim \frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i}). \tag{8}$$

The other special case with $\delta = 0$ has been used in several papers.[40] This special case is characterized by the condition that the decision maker does not have a strict preference for probability mixtures.

Condition (Probability Mixture Neutrality): For all $i \neq 1$ and $x, y \in \mathbb{R}$ such that $x > 0 > y$,

$$(x, (0)_{-i}) \sim (y, (0)_{-i}) \Rightarrow \frac{1}{2}(x, (0)_{-i}) \oplus \frac{1}{2}(y, (0)_{-i}) \sim (x, (0)_{-i}).$$

Proposition 6 *Suppose $\succsim$ is represented by an EIA model with (α, β, δ) where $(\alpha, \beta) \neq \boldsymbol{0}$.*
(i) $\succsim$ exhibits indifference to timing of mixture if and only if $\delta = 1$.
(ii) $\succsim$ exhibits probability mixture neutrality if and only if $\delta = 0$.

Proof: Assume $\alpha_i > 0$ for some $i \neq 1$, without loss of generality. To prove (i), note that $V(\frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i})) = -\frac{1}{2}(1 - \delta)(\alpha_i + \beta_i)$. Hence, $(0, (0)_{-i}) \sim \frac{1}{2}(1, (0)_{-i}) \oplus$

[36] By inequality aversion, $(0, (0)_{-i}) \succsim (1, (0)_{-i})$ and $(0, (0)_{-i}) \succsim (-1, (0)_{-i})$. Hence, by monotonicity, α_i and β_i are nonnegative. We use 1 and -1 just for simplicity. Instead of 1 and -1, we could use any real numbers x and y such that $x > y$. In that case, $\frac{x+y}{2}$ must be used instead of 0.

[37] Any $i \neq 1$ works.

[38] Given (7), the representation shows $c = -\frac{1}{2}(1 - \delta)(\alpha_i + \beta_i)$. For inequality-averse subjects, it is natural to assume $(0, (0)_{-i}) \succsim \frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i})$. Hence, under monotonicity, c is negative. Thus, $\delta \leq 1$.

[39] The characterization of α and β are similar and easier.

[40] For example, Fehr, Klein, and Schmidt (2007) and Grund and Sliwka (2005) used the special case.

$\frac{1}{2}(-1,(0)_{-i}) \Leftrightarrow \delta = 1$, so that (i) holds. To prove (ii), choose any $x, y \in \mathbb{R}$ such that $x \succ 0 \succ y$ and $(x,(0)_{-i}) \sim (y,(0)_{-i})$. Then, $U(x,(0)_{-i}) = -\alpha_i x$ and $U(y,(0)_{-i}) = \beta_i y$. Consider the case where $\frac{1}{2}x + \frac{1}{2}y \geq 0$.[41] Then $V(\frac{1}{2}(x,(0)_{-i}) \oplus \frac{1}{2}(y,(0)_{-i})) - U(x,(0)_{-i}) = \delta\alpha_i\frac{1}{2}(x-y)$. Hence, $\frac{1}{2}(x,(0)_{-i}) \oplus \frac{1}{2}(y,(0)_{-i}) \sim (x,(0)_{-i}) \Leftrightarrow \delta = 0$. For the other case where $\frac{1}{2}x + \frac{1}{2}y \leq 0$, the result can be proved in the same way. ■

The above proposition states that these two special cases have rather extreme implications for behavior. This would suggest that the generic case where $\delta \in (0,1)$ is more relevant.

6 Conclusion

We have introduced a model of inequality aversion of an individual under risk, which we call the EIA model. Under risk, there are two distinct and sometimes incompatible notions of equality, in contrast to a deterministic environment: equality of opportunity and equality of outcome. In the EIA model, a single parameter δ captures a preference for equality of opportunity relative to a preference for equality of outcome. Furthermore, the model reduces into the model of Fehr and Schmidt (1999) on the deterministic allocations. We have applied the EIA model to provide a unified explanation for various experimental evidence, including a preference for efficiency. Moreover, we have applied the EIA model to the optimal tournament and have shown that the efficiency can be achieved as $\delta \to 1$ in spite of inequality aversion. One underling insight suggested by these applications is that equality of opportunity can often be compatible with efficiency in contrast to equality of outcome. Finally, we have provided behavioral conditions that characterize the EIA model.

Appendix: Proofs

Before proving the theorem, we show two independent lemmas. The first lemma provides the representation theorem of the model of Fehr and Schmidt (1999) on the restricted domain $\mathbb{R}^n$ of the set of degenerate lotteries.

[41] In this case, $V(\frac{1}{2}(x,(0)_{-i}) \oplus \frac{1}{2}(y,(0)_{-i})) = -\delta\alpha_i\left[\frac{1}{2}x + \frac{1}{2}y\right] - (1-\delta)\frac{1}{2}\left[\alpha_i x - \beta_i y\right]$.

A Lemmas

Lemma 1 *A preference* $\succsim$ *on* $\mathbb{R}^n$ *satisfies Rationality, Inequality Aversion, and Outcome Quasi-Comonotonic Independence if and only if there exists a pair* (α, β) *of vectors* $\alpha, \beta \in \mathbb{R}_+^{n-1}$ *such that* $\succsim$ *on* $\mathbb{R}^n$ *is represented by* $U(\boldsymbol{x}) = x_1 - \sum_{i=2}^{n} \big(\alpha_i \max\{x_i - x_1, 0\} + \beta_i \max\{x_1 - x_i, 0\}\big)$.

Proof of Lemma 1: The necessity of the conditions is trivial. In the following, we show the sufficiency. Fix $\succsim$ that satisfies the conditions.

Step 1: There exists a utility function $U : \mathbb{R}^n \to \mathbb{R}$ such that (i) for all $\alpha \in [0, 1]$ and $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, if $\boldsymbol{x}$ and $\boldsymbol{y}$ are quasi-comonotonic, then $U(\alpha\boldsymbol{x} + (1-\alpha)\boldsymbol{y}) = \alpha U(\boldsymbol{x}) + (1-\alpha)U(\boldsymbol{y})$, (ii) for all $\boldsymbol{x} \in \mathbb{R}^n$ and $a \in \mathbb{R}_+$, $U(a\boldsymbol{x}) = aU(\boldsymbol{x})$, and (iii) U is unique up to positive affine transformation.

Proof of Step 1: Since an equal allocation $(z, ..., z)$ is quasi-comonotonic with any other allocations, Outcome Quasi-comonotonic Independence implies that for all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, $z \in \mathbb{R}$, and $\alpha \in (0, 1)$, $\boldsymbol{x} \succsim \boldsymbol{y} \Leftrightarrow \alpha\boldsymbol{x} + (1-\alpha)(z, ..., z) \succsim \alpha\boldsymbol{y} + (1-\alpha)(z, ..., z)$. By the standard argument with the von Neumann-Morgenstern's Theorem, the above property implies that there exists a utility function U on $\mathbb{R}^n$ such that for all $\boldsymbol{x} \in \mathbb{R}^n$, $z \in \mathbb{R}$, and $\alpha \in (0, 1)$,

$$U(\alpha\boldsymbol{x} + (1-\alpha)(z, ..., z)) = \alpha U(\boldsymbol{x}) + (1-\alpha)U(z, ..., z). \tag{9}$$

In addition, U is unique up to positive affine transformation, so that (iii) holds. Hence, we can normalize U by $U(1, ..., 1) = 1$ and $U(-1, ..., -1) = -1$, without loss of generality. Then, $U(0, ..., 0) = 0$. Therefore, (9) shows that for all $\boldsymbol{x} \in \mathbb{R}$ and $a \in [0, 1]$ such that $a \leq 1$, $U(a\boldsymbol{x}) = aU(\boldsymbol{x})$. This immediately implies (ii).[42]

To show (i), choose $\alpha \in [0, 1]$ and $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$ such that $\boldsymbol{x}$ and $\boldsymbol{y}$ are quasi-comonotonic. By (ii), there exists a positive integer n such that $\frac{1}{n}U(\boldsymbol{x}) = U(\frac{1}{n}\boldsymbol{x}) \in [-1, 1]$. Since $[-1, 1] \subset \{U(z, ..., z) | z \in \mathbb{R}\}$, there exists $z \in \mathbb{R}$ such that $(z, ..., z) \sim \frac{1}{n}\boldsymbol{x}$. In addition, $\frac{1}{n}\boldsymbol{x}$ and $\frac{1}{n}\boldsymbol{y}$ are also quasi-comonotonic. Hence, Outcome Quasi-comonotonic Independence shows that

[42] Let $a > 1$ and $x \in \mathbb{R}$. Then, $\boldsymbol{x} \sim \frac{1}{a}(a\boldsymbol{x}) + \big(1 - \frac{1}{a}\big)(0, ..., 0)$. Hence, $U(\boldsymbol{x}) = \frac{1}{a}U(a\boldsymbol{x})$, so that $U(a\boldsymbol{x}) = aU(\boldsymbol{x})$.

$\alpha(\frac{1}{n}\boldsymbol{x}) + (1-\alpha)(\frac{1}{n}\boldsymbol{y}) \sim \alpha(z,...,z) + (1-\alpha)(\frac{1}{n}\boldsymbol{y})$. Therefore, by using (9) and (ii), we obtain $\frac{1}{n}U(\alpha\boldsymbol{x}+(1-\alpha)\boldsymbol{y}\big) = U(\alpha(z,...,z)+(1-\alpha)(\frac{1}{n}\boldsymbol{y})) = \alpha U(z,...,z)+(1-\alpha)U(\frac{1}{n}\boldsymbol{y}) = \alpha\big(\frac{1}{n}U(\boldsymbol{x})\big) + (1-\alpha)\big(\frac{1}{n}U(\boldsymbol{y})\big)$, so that (i) holds. □

For all $i \neq 1$, define $\alpha_i = -U(1,(0)_{-i})$ and $\beta_i = -U(-1,(0)_{-i})$. By Inequality Aversion, $\alpha_i \geq 0$ and $\beta_i \geq 0$ for all $i \neq 1$. Fix any $\boldsymbol{x} \in \mathbb{R}^n$.

Step 2: Let $\overline{I} = \{i \in I | x_i > x_1\}$ and $\underline{I} = \{i \in I | x_1 > x_i\}$. Then, (i) for all $i \in \overline{I}$, $U(x_i - x_1,(0)_{-i}) = -\alpha_i \max\{x_i - x_1, 0\}$, (ii) for all $i \in \underline{I}$, $U(x_i - x_1,(0)_{-i}) = -\beta_i \max\{x_1 - x_i, 0\}$, and (iii) for all $i \in I \setminus (\overline{I} \cup \underline{I})$, $U(x_i - x_1,(0)_{-i}) = 0$.

Proof of Step 2: To show (i), fix $i \in \overline{I}$. By Step 1 (ii), $U(x_i - x_1,(0)_{-i}) = (x_i - x_1)U(1,(0)_{-i}) = -\alpha_i \max\{x_i - x_1, 0\}$. (ii) and (iii) can be proved in the same way. □

By the steps, the following calculations complete the proof of Lemma 1. Note that $\frac{1}{n}\boldsymbol{x} \sim \frac{1}{n}(x_1,...,x_1) + \sum_{i=2}^{n} \frac{1}{n}(x_i - x_1,(0)_{-i})$ and any pair among $\{(x_1,...,x_1),(x_2 - x_1,(0)_{-2}),...,(x_n - x_1,(0)_{-n})\}$ is quasi-comonotonic. Therefore, $U(\boldsymbol{x}) = nU(\frac{1}{n}\boldsymbol{x}) = nU(\frac{1}{n}(x_1,...,x_1) + \sum_{i=2}^{n} \frac{1}{n}(x_i - x_1,(0)_{-i})) = U(x_1,...,x_1) + \sum_{i=2}^{n} U(x_i - x_1,(0)_{-i}) = x_1 - \sum_{i=2}^{n}(\alpha_i \max\{x_i - x_1, 0\} + \beta_i \max\{x_1 - x_i, 0\})$. ■

Lemma 2 provides a representation result for a preference $\hat{\succsim}$ on a subset set $\mathscr{D}$ of $\mathbb{R}^2$.

Condition (Certainty Monotonicity): $(c,c) \hat{\succsim} (c',c')$ if and only if $c \geq c'$.

Condition (Certainty Independence): For all $(a,b),(a',b'),(c,c) \in \mathscr{D}$ and $\alpha \in [0,1]$, $(a,b) \hat{\succsim} (a',b')$ if and only if $\alpha(a,b) + (1-\alpha)(c,c) \hat{\succsim} \alpha(a',b') + (1-\alpha)(c,c)$.

Condition (Certainty Continuity): For all $(a,b),(a',b'),(c,c) \in \mathscr{D}$, if $(a,b) \hat{\succsim} (a',b') \hat{\succsim} (c,c)$ or $(c,c) \hat{\succsim} (a',b') \hat{\succsim} (a,b)$ then there exists $\alpha \in [0,1]$ such that $(a',b') \hat{\sim} \alpha(a,b) + (1-\alpha)(c,c)$.

Lemma 2 *: Suppose that (i) for any $(a,b) \in \mathscr{D}$, $a \geq b$, (ii) for all $(a,b),(c,c) \in \mathscr{D}$ and $\alpha \in [0,1]$, $\alpha(a,b) + (1-\alpha)(c,c) \in \mathscr{D}$, and (iii) there exist $(a^*,b^*),(\overline{c},\overline{c}),(\underline{c},\underline{c}) \in \mathscr{D}$ such that $a^* > b^*$ and $\overline{c} > a^* > \underline{c}$. If a preference $\hat{\succsim}$ on $\mathscr{D}$ satisfies Completeness, Transitivity, Certainty Monotonicity, Certainty Independence, and Certainty Continuity, then there exists a real number δ such that $(a,b) \hat{\succsim} (a',b') \Leftrightarrow \delta a + (1-\delta)b \geq \delta a' + (1-\delta)b'$.*

Proof of Lemma 2: Consider the case where $(a^*, a^*) \hat{\succsim} (a^*, b^*)$. By assumption (iii), $\overline{c} > a^*$. Certainty Monotonicity shows $(\overline{c}, \overline{c}) \hat{\succ} (a^*, a^*)$. Then by Certainty Continuity, there exist $\overline{\alpha} > 0$ such that $(a^*, a^*) \hat{\sim} \overline{\alpha}(a^*, b^*) + (1-\overline{\alpha})(\overline{c}, \overline{c})$. Denote $\big(\overline{\alpha}a^* + (1-\overline{\alpha})\overline{c}, \overline{\alpha}b^* + (1-\overline{\alpha})\overline{c}\big)$ by $(\hat{a}, \hat{b})$. [43] Then, $(\hat{a}, \hat{b}) \hat{\sim} (a^*, a^*)$.

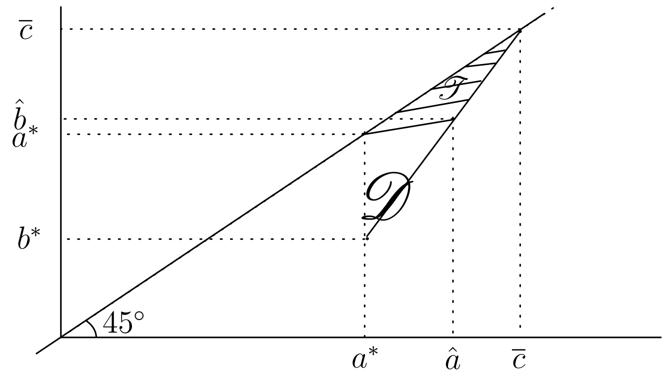


Figure 5: Indifference curves of $\hat{\succsim}$ on $\mathscr{T}$

Let $\mathscr{T}$ be a triangle including the interior which consists of the vertices $(\overline{c}, \overline{c})$, (a^*, a^*), and $(\hat{a}, \hat{b})$.[44] In the following, we show the existence of the desired real number δ on $\mathscr{T}$ and then extend the result on $\mathscr{D}$.

Step 1: $\mathscr{T}$ is a nondegenerate subset of $\mathscr{D}$.

Proof of Step 1: Since $a^* > b^*$ and $\overline{\alpha} > 0$, $\hat{a} > \hat{b}$. Therefore, $(a^*, a^*) \neq (\hat{a}, \hat{b}) \neq (\overline{c}, \overline{c})$. Hence, $\mathscr{T}$ is not degenerate. Choose any $(a, b) \in \mathscr{T}$ to show $(a, b) \in \mathscr{D}$. Since $\mathscr{T}$ is a triangle, there exist $\alpha, \beta \in [0, 1]$ such that $(a, b) = \alpha(\overline{c}, \overline{c}) + \beta(a^*, a^*) + (1 - \alpha - \beta)(\hat{a}, \hat{b}) = (\alpha + \beta)(c, c) + (1 - \alpha - \beta)(\hat{a}, \hat{b})$, where $c \equiv \frac{\alpha}{\alpha+\beta}\overline{c} + \frac{\beta}{\alpha+\beta}a^*$. Since $(\hat{a}, \hat{b}) \in \mathscr{D}$, it follows from assumption (ii) that $(a, b) \in \mathscr{D}$. ■

Step 2: There exists a real number δ such that for any $(a, b), (a', b') \in \mathscr{T}$, $(a, b) \hat{\succsim} (a', b') \Leftrightarrow \delta a + (1 - \delta)b \geq \delta a' + (1 - \delta)b'$.

Proof of Step 2:

[43] In the other case where $(a^*, b^*) \hat{\succsim} (a^*, a^*)$, there exist $\underline{\alpha} > 0$ such that $(a^*, a^*) \hat{\sim} \underline{\alpha}(a^*, b^*) + (1 - \underline{\alpha})(\underline{c}, \underline{c})$. Denote $\big(\underline{\alpha}\underline{c} + (1 - \underline{\alpha})a^*, \underline{\alpha}\underline{c} + (1 - \underline{\alpha})b^*\big)$ by $(\tilde{a}, \tilde{b})$. Then, instead of the triangle $\mathscr{T}$ defined in the proof, consider a triangle including the interior, which consists of the vertices $(\tilde{c}, \tilde{c})$, $(\underline{c}, \underline{c})$, and (a^*, a^*). Then, the rest of the proof goes through exactly in the same way.

[44] Formally, $\mathscr{T} = \big\{(a, b) \in \mathbb{R}^2 | a \geq b, \big\langle (a^* - \hat{a}, \hat{b} - a^*), (a, b) - (a^*, a^*) \big\rangle \geq 0, \big\langle (\overline{c} - \hat{a}, \hat{b} - \overline{c}),\ (a, b) - (\overline{c}, \overline{c}) \big\rangle \geq 0 \big\}$, where $\langle \cdot, \cdot \rangle$ is a inner product.

Substep 2.1: For all $(a,b) \in \mathscr{T}$, there exists a unique number $\alpha \in [0,1]$ such that $(a,b) \hat{\sim} \alpha(\bar{c},\bar{c}) + (1-\alpha)(a^*,a^*)$.

Proof of Substep 2.1: Choose any $(a,b) \in \mathscr{T}$. Since $\mathscr{T}$ is the triangle, there exist $\alpha, \beta \in [0,1]$ such that $(a,b) = \alpha(\bar{c},\bar{c}) + \beta(a^*,a^*) + (1-\alpha-\beta)(\hat{a},\hat{b})$. Since $(\hat{a},\hat{b}) \hat{\sim} (a^*,a^*)$, Transitivity and Certainty Independence show $(a,b) \hat{\sim} \alpha(\bar{c},\bar{c}) + (1-\alpha)(a^*,a^*)$. Since $\bar{c} > a^*$, Certainty Monotonicity shows that α is unique. Hence, Substep 2.1 is proved.

For all $(a,b) \in \mathscr{T}$, define $c(a,b) = \alpha\bar{c} + (1-\alpha)a^*$, where α is obtained in Substep 2.1.

Substep 2.2: For all $(a,b) \in \mathscr{T}$, $\dfrac{a - c(a,b)}{a-b} = \dfrac{\hat{a} - a^*}{\hat{a} - \hat{b}}$.

Proof of Substep 2.2: Choose any $(a,b) \in \mathscr{T}$. Since $c(a,b) = \alpha\bar{c} + (1-\alpha)a^*$, $\frac{a-c(a,b)}{a-b} = \frac{a-\alpha\bar{c}-(1-\alpha)a^*}{a-b}$. Since $(a,b) \in \mathscr{T}$, $(a,b) = \alpha'(\bar{c},\bar{c}) + \beta(a^*,a^*) + (1-\alpha'-\beta)(\hat{a},\hat{b})$ for some $\alpha', \beta \in [0,1]$. Since $(a^*,a^*) \hat{\sim} (\hat{a},\hat{b})$, Transitivity and Certainty Independence show $(a,b) \hat{\sim} \alpha'(\bar{c},\bar{c}) + (1-\alpha')(a^*,a^*)$. Hence, Substep 2.1 shows $\alpha' = \alpha$, so that $(a,b) = \alpha(\bar{c},\bar{c}) + \beta(a^*,a^*) + (1-\alpha-\beta)(\hat{a},\hat{b})$. Thus, $a - \alpha\bar{c} - (1-\alpha)a^* = (\alpha\bar{c} + \beta a^* + (1-\alpha-\beta)\hat{a}) - \alpha\bar{c} - (1-\alpha)a^* = (1-\alpha-\beta)(\hat{a}-a^*)$ and $a - b = (1-\alpha-\beta)(\hat{a}-\hat{b})$. Hence, $\frac{a-\alpha\bar{c}-(1-\alpha)a^*}{a-b} = \frac{\hat{a}-a^*}{\hat{a}-\hat{b}}$, so that Substep 2.2 is proved.

Define $\delta = 1 - \dfrac{\hat{a}-a^*}{\hat{a}-\hat{b}}$. By substituting this into the result of Substep 2.2, we conclude that for all $(a,b) \in \mathscr{T}$, $c(a,b) = \delta a + (1-\delta)b$. Therefore, for any $(a,b),(a',b') \in \mathscr{T}$ by Substeps 2.1 and 2.2, $(a,b) \hat{\succsim} (a',b') \Leftrightarrow (c(a,b),c(a,b)) \hat{\succsim} (c(a',b'),c(a',b')) \Leftrightarrow c(a,b) \geq c(a',b') \Leftrightarrow \delta a + (1-\delta)b \geq \delta a' + (1-\delta)b'$. ■

Step 3: For all $(a,b),(a',b') \in \mathscr{D}$, $(a,b) \hat{\succsim} (a',b') \Leftrightarrow \delta a + (1-\delta)b \geq \delta a' + (1-\delta)b'$.

Proof of Step 3: Let $c^* = \frac{1}{2}\bar{c} + \frac{1}{2}a^*$. Choose any $(a,b),(a',b') \in \mathscr{D}$. Then, by assumption (i), $a \geq b$ and $a' \geq b'$. Thus, there exists $\alpha \in (0,1]$ such that $\alpha(a,b) + (1-\alpha)(c^*,c^*)$ and $\alpha(a',b') + (1-\alpha)(c^*,c^*)$ belong to $\mathscr{T}$. Therefore, Step 2 and Certainty Independence show that $(a,b) \hat{\succsim} (a',b') \Leftrightarrow \alpha(a,b) + (1-\alpha)(c^*,c^*) \hat{\succsim} \alpha(a',b') + (1-\alpha)(c^*,c^*) \Leftrightarrow \delta(\alpha a + (1-\alpha)c^*) + (1-\delta)(\alpha b + (1-\alpha)c^*) \geq \delta(\alpha a' + (1-\alpha)c^*) + (1-\delta)(\alpha b' + (1-\alpha)c^*) \Leftrightarrow \delta a + (1-\delta)b \geq \delta a' + (1-\delta)b'$. ■

B Proof of Theorem and Proposition 1

The necessity of the conditions is trivial. To show Continuity, note that the EIA model is a weighted sum of max functions. In the following, we show the sufficiency. Fix $\succsim$ on $\Delta(\mathbb{R}^n)$

that satisfies the conditions.

Step 1: There exists a utility function V such that (i) $V(\alpha p \oplus (1-\alpha)(x, ..., x)) = \alpha V(p) + (1-\alpha)V(x, ..., x)$ for all $p \in \Delta(\mathbb{R}^n)$, $x \in \mathbb{R}$, and $\alpha \in [0, 1]$; (ii) $V = U$ on $\mathbb{R}^n$, where U is a degenerate EIA model.

Proof of Step 1: By Lemma 1, there exists a degenerate EIA model U of $\succsim$ on $\mathbb{R}^n$. As in Step 1 of the proof of Lemma 1, there exists a utility function V such that for all $p \in \Delta(\mathbb{R}^n)$, $x \in \mathbb{R}$, and $\alpha \in [0, 1]$, $V(\alpha p \oplus (1-\alpha)(x, ..., x)) = \alpha V(p) + (1-\alpha)V(x, ..., x)$, so that (i) holds. In addition, V is unique up to positive affine transformation. Hence, we can normalize V by $V(1, ..., 1) = 1$ and $V(-1, ..., -1) = -1$, without loss of generality.

To prove (ii), it suffices to show that $V = U$ on $\{(x, ..., x)|x \in \mathbb{R}\}$ because V is unique up to positive affine transformation. Since $U(x, ..., x) = x$ for all $x \in \mathbb{R}$, we prove $V(x, ..., x) = x$ for all $x \in \mathbb{R}$. Note that for all $x, y \in \mathbb{R}$, $V(x, ..., x) \geq V(y, ..., y) \Leftrightarrow (x, ..., x) \succsim (y, ..., y) \Leftrightarrow x \geq y$. Hence, there exists an increasing function $f : \mathbb{R} \to \mathbb{R}$ such that $V(x, ..., x) = f(x)$ for all $x \in \mathbb{R}$. To show that f is the identity function, note that Dominance trivially implies $\alpha(x, ..., x) + (1-\alpha)(y, ..., y) \sim \alpha(x, ..., x) \oplus (1-\alpha)(y, ..., y)$ for all $x, y \in \mathbb{R}$ and $\alpha \in [0, 1]$. Therefore, property (i) of V shows that f is affine (i.e, $f(\alpha x + (1-\alpha)y) = \alpha f(x) + (1-\alpha)f(y)$).[45] In addition, by the normalizations, $f(1) = 1$ and $f(-1) = -1$. Therefore, f is the identity function. ■

If $(\alpha, \beta) = \mathbf{0}$, then the theorem holds trivially. Thus, we consider the case where $(\alpha, \beta) \neq \mathbf{0}$. To use Lemma 2, define $\mathscr{D} = \left\{ \left(U(E_p(\boldsymbol{x})),\ E_p(U(\boldsymbol{x})) \right) \in \mathbb{R}^2 \;\middle|\; p \in \Delta(\mathbb{R}^n) \right\}$.

Step 2: $\mathscr{D}$ satisfies the properties (i), (ii), and (iii) in Lemma 2.

Proof of Step 2: To show (i), it suffices to show that $U(E_p(\boldsymbol{x})) \geq E_p(U(x))$ for all $p \in \Delta(\mathbb{R}^n)$. To show this, fix $p \in \Delta(\mathbb{R}^n)$. For all $i \neq 1$ and $\boldsymbol{x} = (x_1, ..., x_n) \in \mathbb{R}^n$, define $G_i(\boldsymbol{x}) = -\alpha_i \max\{x_i - x_1, 0\} - \beta_i \max\{x_1 - x_i, 0\}$. Then, G_i is concave and $U(\boldsymbol{x}) = x_1 + \sum_{i \neq 1} G_i(\boldsymbol{x})$ for all $\boldsymbol{x} \in \mathbb{R}^n$.[46] Since U is a sum of concave functions, U is also concave. Therefore, Jensen's

[45] To see this fix $x, y \in \mathbb{R}$ and $\alpha \in [0, 1]$. Then $\alpha f(x) + (1-\alpha)f(y) = \alpha V(x, ..., x) + (1-\alpha)V(y, ..., y) = V(\alpha(x, ..., x) \oplus (1-\alpha)(y, ..., y)) = V(\alpha(x, ..., x) + (1-\alpha)(y, ..., y)) = f(\alpha x + (1-\alpha)y)$. Hence, f is affine.

[46] Remember (i) for any convex function ϕ and positive number a, $a \max\{\phi(\cdot), 0\}$ is convex; (ii) a sum of convex functions is convex; (iii) for any convex function ϕ, $-\phi$ is concave. By (i), $\alpha_i \max\{x_i - x_1, 0\}$ and $\beta_i \max\{x_1 - x_i, 0\}$ are convex. Then, by (ii), $\alpha_i \max\{x_i - x_1, 0\} + \beta_i \max\{x_1 - x_i, 0\}$ is convex. Finally, then by (iii), G_i is concave.

Inequality shows that $U(E_p(\boldsymbol{x})) \geq E_p(U(\boldsymbol{x}))$ for all $p \in \Delta(\mathbb{R}^n)$. Property (ii) follows from property (9) of U in Lemma 1. To show (iii) assume $\alpha_i > 0$ for some $i \neq 1$ without loss of generality. Let $p = \frac{1}{2}(1,(0)_{-i}) \oplus \frac{1}{2}(-1,(0)_{-i})$. Define $(a^*, b^*) = (U(E_p(\boldsymbol{x})), E_p(U(\boldsymbol{x})))$. Then $a^* = U(\frac{1}{2}(1,(0)_{-i}) + \frac{1}{2}(-1,(0)_{-i})) = 0 > -\frac{1}{2}(\alpha_i + \beta_i) = \frac{1}{2}U(1,(0)_{-i}) + \frac{1}{2}U(-1,(0)_{-i}) = b^*$. Obviously, there exist $\bar{c}, \underline{c} \in \mathbb{R}$ such that $\bar{c} > a^*$ and $b^* > \underline{c}$. This completes the proof of (iii). ■

For all $(a,b) \in \mathscr{D}$, define $v(a,b) = V(p)$, where $p \in \Delta(\mathbb{R}^n)$ such that $U(E_p(\boldsymbol{x})) = a$ and $E_p(U(\boldsymbol{x})) = b$. Dominance implies that v is a well-defined increasing function.[47] Then, for all $(a,b),(a',b') \in \mathscr{D}$ define, $(a,b) \hat{\succsim} (a',b') \Leftrightarrow v(a,b) \geq v(a',b')$.

Step 3: $\hat{\succsim}$ satisfies Completeness, Transitivity, Certainty Monotonicity, Certainty Independence, and Certainty Continuity.

Proof of Step 3: Since v is a well-defined increasing function, $\hat{\succsim}$ satisfies Completeness, Transitivity, and Certainty Monotonicity. Certainty Independence follows from Step 1. To see this, choose any $(a,b),(a',b'),(z,z) \in \mathscr{D}$ and $\alpha \in [0,1]$. Then, there exist $p, q \in \Delta(\mathbb{R}^n)$ such that $(a,b) = \big(U(E_p(\boldsymbol{x})), E_p(U(\boldsymbol{x}))\big)$ and $(a',b') = \big(U(E_q(\boldsymbol{x})), E_q(U(\boldsymbol{x}))\big)$. To simplify notation, let $p' = \alpha p \oplus (1-\alpha)(z,...,z)$ and $q' = \alpha q \oplus (1-\alpha)(z,...,z)$. By Step 1(i), then $(a,b) \hat{\succsim} (a',b') \Leftrightarrow V(p) \geq V(q) \Leftrightarrow V(p') \geq V(q') \Leftrightarrow \big(U(E_{p'}(\boldsymbol{x})), E_{p'}(U(\boldsymbol{x}))\big) \hat{\succsim} \big(U(E_{q'}(\boldsymbol{x})), E_{q'}(U(\boldsymbol{x}))\big)$. Moreover, since $\big(U(E_{p'}(\boldsymbol{x})), E_{p'}(U(\boldsymbol{x}))\big) = \alpha(a,b) + (1-\alpha)(z,z)$ and $\big(U(E_{q'}(\boldsymbol{x})), E_{q'}(U(\boldsymbol{x}))\big) = \alpha(a',b') + (1-\alpha)(z,z)$. This completes the proof of Certainty Independence.

Finally, we show that $\hat{\succsim}$ satisfies Certainty Continuity. Choose any $(a,b),(a',b'),(z,z) \in \mathscr{D}$ such that $(a,b) \hat{\succsim} (a',b') \hat{\succsim} (z,z)$. Then, there exist $p, q \in \Delta(\mathbb{R}^n)$ such that $\big(U(E_p(\boldsymbol{x})), E_p(U(\boldsymbol{x}))\big) = (a,b)$ and $\big(U(E_q(\boldsymbol{x})), E_q(U(\boldsymbol{x}))\big) = (a',b')$. Then $p \succsim q \succsim (z,...,z)$. Then by Continuity, there exists $\alpha \in [0,1]$ such that $q \sim \alpha p \oplus (1-\alpha)(z,...,z)$. By definition, this implies that $(a',b') \hat{\sim} \alpha(a,b) + (1-\alpha)(z,z)$. The other case where $(z,z) \hat{\succsim} (a',b') \hat{\succsim} (a,b)$ is proved in the same way. ■

Given Steps 2 and 3, Lemmas 2 shows that $p \succsim q \Leftrightarrow V(p) \geq V(q) \Leftrightarrow \big(U(E_p(\boldsymbol{x})), E_p(U(\boldsymbol{x}))\big)$

[47] To see that v is well-defined (i.e., if $(U(E_p(\boldsymbol{x})), E_p(U(\boldsymbol{x}))) = (U(E_q(\boldsymbol{x})), E_q(U(\boldsymbol{x})))$, then $V(p) = V(q)$), choose any $p, q \in \Delta(\mathbb{R}^n)$ such that $(U(E_p(\boldsymbol{x})), E_p(U(\boldsymbol{x}))) = (U(E_q(\boldsymbol{x})), E_q(U(\boldsymbol{x})))$. Then, conditions (i) and (ii) in Dominance are satisfied with equality. Therefore, Dominance shows $V(p) = V(q)$.

$\hat{\succsim} \big(U(E_q(\boldsymbol{x})), E_q(U(\boldsymbol{x}))\big) \Leftrightarrow \delta U(E_p(\boldsymbol{x})) + (1-\delta)E_p(U(\boldsymbol{x})) \geq \delta U(E_q(\boldsymbol{x})) + (1-\delta)E_q(U(\boldsymbol{x}))$ for all $p, q \in \Delta(\mathbb{R}^n)$. This completes the proof of Theorem.

In the following, we provide the proof of Proposition 1

Proof: It is easy to see that (ii) implies (i). To show that (i) implies (ii), suppose that (δ, α, β) and $(\delta', \alpha', \beta')$ represent $\succsim$. Let V and V' be the function defined by (δ, α, β) and $(\delta', \alpha', \beta')$, respectively.

First, we show $(\alpha, \beta) = (\alpha', \beta')$. Suppose to the contrary that $(\alpha, \beta) \neq (\alpha', \beta')$. Assume $\alpha'_i > \alpha_i$ for some $i \neq 1$, without loss of generality. Therefore, $V(1, (0)_{-i}) = -\alpha_i = V(-\alpha_i, ..., -\alpha_i) = -\alpha_i > -\alpha'_i = V(-\alpha'_i, ..., -\alpha'_i)$ and $V'(1, (0)_{-i}) = -\alpha'_i = V'(-\alpha'_i, ..., -\alpha'_i)$. Thus, $(1, (0)_{-i}) \sim (-\alpha_i, ..., -\alpha_i) \succ (-\alpha'_i, ..., -\alpha'_i) \sim (1, (0)_{-i})$, which is a contradiction.

Second, we show that if $(\alpha, \beta) \neq \mathbf{0}$, then $\delta = \delta'$. Suppose to the contrary that $(\alpha, \beta) \neq 0$ but $\delta \neq \delta'$. Without loss of generality, assume $\delta > \delta'$ and $\alpha_i > 0$ for some $i \neq 1$. Then, $V(\frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i})) = -\frac{1}{2}(1-\delta)(\alpha_i + \beta_i) = V(-\frac{1}{2}(1-\delta)(\alpha_i + \beta_i), ..., -\frac{1}{2}(1-\delta)(\alpha_i + \beta_i))$. Therefore, $\frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i}) \sim (-\frac{1}{2}(1-\delta)(\alpha_i + \beta_i), ..., -\frac{1}{2}(1-\delta)(\alpha_i + \beta_i)) \succ (-\frac{1}{2}(1-\delta')(\alpha_i + \beta_i), ..., -\frac{1}{2}(1-\delta')(\alpha_i + \beta_i)) \sim \frac{1}{2}(1, (0)_{-i}) \oplus \frac{1}{2}(-1, (0)_{-i})$, which is a contradiction.

References

[1] Andreoni, J. and J. Miller (2002): "Giving According to GARP: An Experimental Test of the Consistency of Preferences for Altruism," *Econometrica*, 70, 737–753.

[2] Anscombe, F. J. and R. J. Aumann (1963): "A Definition of Subjective Probability," *The Annals of Mathematical Statistics*, 34, 199–205.

[3] Bertola, G. and A. Ichino (1995): "Wage Inequality and Unemployment: United States vs. Europe," *NBER Macroeconomics Annual,* 10, 13–66.

[4] Bohnet, I., F. Greig, B. Herrmann, and R. Zeckhauser (2008): "Betrayal Aversion: Evidence from Brazil, China, Oman, Switzerland, Turkey, and the United States," *American Economic Review,* 98, 294–310.

[5] Bolton, G and A. Ockenfels (2000): "ERC: A Theory of Equity, Reciprocity, and Competition," *American Economic Review*, 90, 166–193.

[6] Bolton, G, J. Brandts and A. Ockenfels (2005): "Fair Procedures: Evidence from Games Involving Lotteries," *The Economic Journal*, 115, 1054–1076.

[7] Bolton, G. and A. Ockenfels (2006): "Inequality Aversion, Efficiency, and Maxmin Preferences in Simple Distribution Experiments: Comment, " *American Economic Review,* 96, 1906–1911.

[8] Bolton, G. and A. Ockenfels, Forthcoming: "Risk Taking and Social Comparison," *American Economic Review.*

[9] Borah, A (2011): "Other-Regarding Preferences and Concerns for Procedure," Mimeo.

[10] Camerer, C. F. (2003): *Behavioral Game Theory.* Princeton, NJ: Princeton University Press.

[11] Charness, G. and M. Rabin (2002): "Understanding Social Preference with Simple Tests," *The Quarterly Journal of Economics,* 117, 817–869.

[12] Cox, J. and V. Sadiraj (2004): "Direct Tests of Social Preferences and a New Model," University of Arizona, Mimeo.

[13] Diamond, P. A. (1967): "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparison of Utility: Comment," *The Journal of Political Economy*, 75, 765–766.

[14] Engelmann, D. and M. Strobel (2004): "Inequality Aversion, Efficiency, and Maxmin Preferences in Simple Distribution Experiments," *American Economic Review*, 94, 857–869.

[15] Engelmann, D. and M. Strobel (2006): "Inequality Aversion, Efficiency, and Maxmin Preferences in Simple Distribution Experiments: Reply," *American Economic Review*, 96, 1918–1923.

[16] Fehr, E. and K. M. Schmidt (1999): "A Theory of Fairness, Competition, and Cooperation," *The Quarterly Journal of Economics,* 114, 817–868.

[17] Fehr, E. and K. M. Schmidt (2005): "The Economics of Fairness, Reciprocity and Altruism – Experimental Evidence and New Theories," in *Handbook on the Economics of Giving, Altruism and Reciprocity*, ed. by S. Kolm and J. M. Ythier. Elsevier, 615–691.

[18] Fehr, E. and K. M. Schmidt (2006): "Inequality Aversion, Efficiency, and Maxmin Preferences in Simple Distribution Experiments: Comment, " *American Economic Review,* 96, 1912–1917.

[19] Fehr, E., A. Klein, and K. M. Schmidt (2007): "Fairness and Contract Design, " *Econometrica,* 75, 121–154.

[20] Fudenberg, D. and D. K. Levine (2012): "Fairness, Risk Preferences and Independence: Impossibility Theorems," *Journal of Economic Behavior and Organization,* 81, 606–612.

[21] Gajdos, T. and E. Maurin (2004): "Unequal Uncertainties and Uncertain Inequalities: An Axiomatic Approach," *Journal of Economic Theory*, 116, 93–118.

[22] Grund, C. and D. Sliwka (2005): "Envy and Compassion in Tournaments," *Journal of Economics and Management Strategy,* 14, 187–207.

[23] Harris, J. (1975): " The Survival Lottery, " *Philosophy*, 50, 81–87.

[24] Kariv, S. and B. Zame (2009), University of California Berkeley, work in progress.

[25] Karni, E., T. Salmon, and B. Sopher (2008):"Individual Sense of Fairness: An Experimental Study," *Experimental Economics*, 11, 174–189.

[26] Kircher, P., S. Luding, and A. Sandroni (2009): "Fairness: A Critique to the Utilitarian Approach," Northwestern University, Mimeo.

[27] Lazer, E. P. and S. Rosen (1981): "Rank-Order Tournament as Optimum Labor Contracts," *Journal of Political Economy*, 89, 841–864.

[28] Machina, M. (1989): "Dynamic Consistency and Non-Expected Utility Models of Choice under Uncertainty," *Journal of Economic Literature*, XXVII, 1622–1668.

[29] Oosterbeek, H., R. Sloof, and G. van de Kuilen (2004): "Cultural Differences in Ultimatum Game Experiments: Evidence from a Meta-Analysis, " *Experimental Economics*, 7, 171–188.

[30] Prescott. E. C. (2004): "Why Do Americans Work so much more than Europeans?" *Federal Reserve Bank of Minneapolis Quarterly Review*, 28, 2–13.

[31] Rohde K. I. M. (2010): "A Preference Foundation for Fehr and Schmidt's Model of Inequality Aversion" *Social Choice and Welfare*, 34, 537–547.

[32] Schmeidler, D. (1989): "Subjective Probability and Expected Utility without Additivity,"*Econometrica*, 57, 571–587.